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Convex Optimization in Power Distribution Networks

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Abstract: *The optimal power flow (OPF) problem is a critical problem for power generation and is generally non-convex. This paper mainly focuses on the tree topology of the distribution networks. It studies the two-bus network with fixed and variable voltage magnitudes. Simulation has been done in Matlab. The result shows that the OPF problem can be treated as a convex problem if one or more voltage magnitudes is fixed in a two-bus network.*
Keywords: *Index Terms—convex optimization, OPF, tree networks*

I. Nomenclature

V: complex voltage vector at all buses.
|V|: voltage magnitude vector at all buses.
θ: voltage angle vector at all buses Y: admittance matrix
g: shunt conductance
b: shunt susceptance
S: vector of apparent power flow on the line
P: vector of active power generations
Q: vector of reactive power generations
L: loss of the line
( )*: Hermitian transpose of a matrix j: the imaginary unit.
Eig: eigenvalue of a matrix
Re: real part of a complex matrix
Im: imaginary part of a complex matrix

II. Introduction

The optimal power flow problem was first discussed in Carpentier'spaper in 1962. The objective of an Optimal Power Flow (OPF) algorithm is to find an optimal operating point, which minimizes the generation cost or network loss, subject to a wide range of practical constraints, e.g. bus voltage limits, bus power limits, thermal line constraints, etc.

The OPF problem is a non-convex and challenging for the following two reasons [1]. Firstly, since the injected power at buses depends quadratically one the voltages at the buses, the optimization problem is non-linear. Secondly, power system need to satisfy a series of constraints such as active/reactive power balance equations, power flow limit of line, bus voltage magnitude limits and active/reactive power generation limits. Given the practical importance of the problem, a great many studies have been developed to give efficient solution methods, including linear programming, non-linear programming, quadratic programming, interior point methods, Lagrangian relaxation, artificial intelligence, fuzzy logic, evolutionary programming, genetic algorithm and particle swarm optimization [2], [3].

One widely used for method is the DC (direct current)-OPF, which linearized the OPF problem with assumptions that the power line is lossless, the voltage magnitudes are fixed and the voltage angles are small [4]. This method is not accurate and will perform poorly if the resistance/inductance ratio of the line is high.

In an effort to convexity the AC OPF problem, various convex relaxation techniques have been developed. Semidefinite programming (SDP) method can create a convex relaxation of the OPF problem. In [5], it proposes solving the Lagrangian dual problem instead of solving the OPF problem directly. The paper proves that its SDP formulation will satisfy a condition ensuring zero duality gap between the primal and dual objective function for most OPF problems. In [6], it shows that the load flow problem of a radial distribution system (tree networks) is a convex problem and can be modeled in the form of a conic program. However, the result could not be applied to a meshed network. Then the question lies on what kind of networks can the OPF problem be convexified.

Power system consists of transmission networks and distribution networks. The transmission network is usually made up of high to very high voltage lines that designed to transfer power from major generators to areas in need, the networks' voltages are typically above 100 kV. Distribution networks is designed to distribute power from the transmission network to end users, it is usually made up of low voltage lines with voltage magnitudes below 100 kV. Traditionally OPF problem mainly focus on transmission networks, but nowadays with increasing interest on renewable energy, distributed generation and smart grid, comes with increasing demand on solving the OPF problem in distribution networks. This paper will focuses on topics about convex optimization in distribution networks.

There are typically two types of distribution networks, radial (tree network) or interconnected network. A tree network leaves the station and passes with no normal connection to any other supply. This is typical of long rural lines. An interconnected network is generally found in urban areas and has multiple connections to other points of supply. Since most distribution networks is with a tree topology and research on tree networks will shed light on the general problem, the goal of this paper is to study on the tree topology of distribution networks.

The paper is organized as follows. In Section I, we establish the notations. In Section III, we state the model used in this paper. In Section IV and Section V, models of a two-bus network with fixed voltage and variable voltage have been studied. In Section VI, we try to explore the possibility of using SDP method to solve OPF problem from anther sight, by checking the Lagrangian dual problem instead of solving OPF directly. And Section VII concludes the paper.

III. Problem Formulation

A. Admittance Matrix

Consider an AC power system with n nodes (buses) and define the network as a graph with the nodes set $[n] = \{1, 2,\ldots, n\}$. We write $i \sim k$ if there is an edge between nodes $i$and k, which means buses i and k are connected. Let y_{ii}denote the shunt admittance-to-ground at bus i, and $y_{ik}= g_{ik}- jb_{ik}$ denote the admittance of the line is buses I and j are connected, where g_{ik}, b_{ik}>0 (the lines are resistive and inductive). The n x n admittance matrix Y can be defined as

$$Y_{ik} = \begin{cases} \sum_{k\sim i} y_{ik} + y_{ii}, & \text{if } i = k \\ -y_{ik}, & \text{if } i \sim k \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Note that this matrix is symmetric but not necessarily Hermitian.

Let the n-dimensional vector of complex bus voltage be $V = (V_1, V_2, \ldots ,V_n)$ and the vector of complex currents be $I = (I_1, I_2, \ldots I_n)$, where V_i is the voltage at bus $i$and $I_i$is the total current flowing out of bus$i$ to the rest of the network. The above parameters are related by Kirchoff's Current Law, $I = YV$.

The vector of complex (apparent) power at bus$i$ is equal to $S_i = P_i + jQ_i = V_i I_i^*$, $i\in [n]$,where P_i and Q_i represent the net active and reactive powers at bus $i$respectively.

B. Classical OPF Problem

Denote G the set of generator buses, L the set of all lines. Let $P_{Di} + jQ_{Di}$ be the active and reactive power at bus i, $i\in [n]$, $V_i= V_{di} + jV_{qi}$ be the voltage vector in rectangular coordinates at each bus i, $i\in [n]$. Let $P_{Gi} + jQ_{Gi}$ be the generator power at generator buses $i\in G$, S_{ik} be the apparent power flow on line $(i, k) \in L$. The upper and lower bounds are expressed by superscripts "max" and "min".

Cost functions associated with each generator $i\in G$ can be expressed as a quadratic objective function as below.

$$fi\,(P_{Gi}) = c_{i2}P^2_{Gi} + c_{i1}P_{Gi} + c_{i0} \quad (2)$$

where c_{i2}, c_{i1}, and c_{i0} are non-negative numbers.

The classical OPF problem can be expressed as below.

$$\min \sum_{k\in G} f_i(P_{Gi})$$

subject to

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \forall\, k \in G \quad (3)$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \forall k \in G \quad (4)$$

$$(V_i^{min})^2 \leq V_{di}^{\,2} + V_{qi}^{\,2} \leq (V_i^{max})^2 \,\forall\, k \in [n] \quad (5)$$

$$|S_{ik}| \leq S_{ik}max \,\forall\, (i,k) \in L \quad (6)$$

$$PGi - PDi = Vd\textstyle\sum i_{t=1}^{n}\left(G_{ti}V_{dt} - B_{ti}V_{qt}\right) + V_{qi}\sum_{t=1}^{n}(B_{ti}V_{dt} + G_{ti}V_{qt}) \quad (7)$$

$$QGi - Q_{Di} = V_{di}\sum_{t=1}^{n}\left(-B_{ti}V_{dt} - G_{ti}V_{qt}\right) + V_q\sum_{i t=1}^{n}(G_{ti}V_{dt} - B_{ti}V_{qt}) \quad (8)$$

The inequalities (3), (4), (5) and (6) limit the power and voltage variables to within the given bounds. The equations (7) and (8) are the physical constrains imposed by the power network.

IV. Case Studies Of Two-Bus Network

Definition of Pareto-front: points on the boundary of the region for which one cannot decrease any component without increasing another component. OPF problems with increasing objective functions defined on the injection region must lie there [8].

A. Two-Bus Network With Angle, Thermal and Flow
Constraints (*Fixed Voltage Magnitudes*).

Figure 1 shows the two-bus network with line admittance $y = g-jb$.

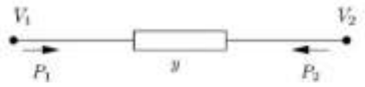


Figure 1. A two-bus network

Let $V_1=|V_1|\exp(j\theta_1)$, $V_2=|V_2|\exp(j\theta_2)$. In this model, assume that $|V_1|$ and $|V_2|$ are fixed, θ_1 and θ_2 can vary. The power flowing from bus 1 to bus 2 can be written as,

$$S_{12}=V_1|V_1-V_2|^* y_{12}^* \quad (9)$$

The power injections at the two buses can be expressed as [8].

$$P_1 = |V_1|^2g+|V_1||V_2|b\sin(\theta)- |V_1||V_2|g\cos(\theta) \quad (10)$$

$$P_2 = |V_2|^2g - |V_1||V_2|b\sin(\theta)- |V_1||V_2|g\cos(\theta) \quad (11)$$

Where $\theta =\theta_1-\theta_2$, $P_1=P_{12}$, $P_2=P_{21}$.

Since the voltage magnitudes $|V_1|$ and $|V_2|$ are fixed, the power flows between two nodes is a function with only one parameter θ.

Mapping equation (10) and (11) on coordinate leads to a hollow ellipse which is centered at ($|V_1|^2g$, $|V_2|^2g$). The ellipse's major axis has a length of $|V_1||V_2|b$, while the minor principle axis has a length of $|V_1||V_2|g$. The major axis is -45 ° to the x-axis.

From [9], we know that in typical power systems: *1)* the inductive reactance of the series elements is much larger than the resistance, and

2) the capacitive susceptance of the shunt elements is much larger than the shunt conductance.

Therefore, for a line between buses i and k,

$$|g_{ik}|<<|b_{ik}| \quad (12)$$

In practice, b/g ratio is larger than 10 for transmission networks and between 3 and 5 for distribution networks [10], [11]. In this paper, we only consider the distribution networks.

Take b=5, g=1, $|V_1|=|V_2|$, simulate equation (10) and (11) in Matlab. The new formulas turned to be

$$P_1 = 1+5\sin(\theta)- \cos(\theta) \quad (13)$$

$$P_2 = 1- 5\sin(\theta) - \cos(\theta) \quad (14)$$

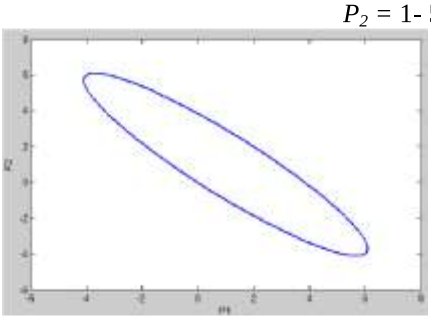


Figure 2. The region corresponding to b=5, g=1, $|V_1|=|V_2|$

The simulation result is shown in figure 2.This reveals the relationship of P_1 and P_2 with only voltage constraints. The bold curve is the Pareto-front, the filled ellipse is the convex hull. It is obvious that the Pareto-fronts of the empty and filled ellipse are the same. Thus, though the empty ellipse is non-convex, it can be treated as a convex optimization problem if the objective function is increasing.

We then consider the effect of thermal, line flow and angle constraints on this problem.

The loss of the line L_{12} can be calculated as

$$L_{12}=P_{12}+P_{21}=|V_1|^2g - 2|V_1||V_2|g\sin(\theta)+ |V_2|^2g \quad (15)$$

Since $|V_1|$ and $|V_2|$ are fixed, the thermal loss and line flow constraints can be translated to angle constraints $\underline{\theta} \leq \theta \leq \overline{\theta}$ for $\underline{\theta} \in [-\pi, 0], \overline{\theta} \in [0, \pi]$. The bold curve in Figure 3 shows the region with a certain angle constraint. The bold curve is the corresponding Pareto-front, and it is obvious that this Pareto-front is the same as that of the convex hull. Thus, though this problem is non-convex, we can solve it as a convex problem with an increasing function *f*.

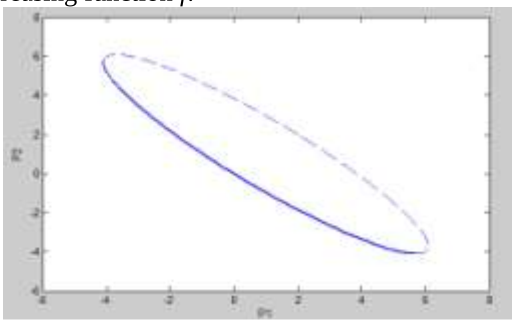


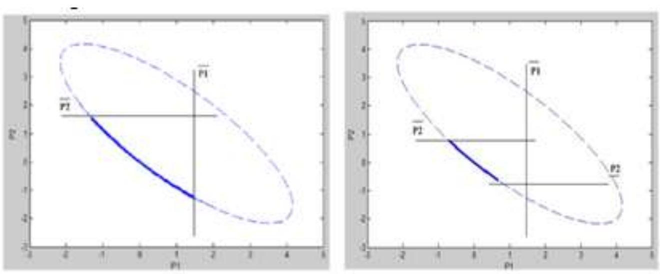
Figure 3. The region corresponding to b=5, g=1, $|V_1|=|V_2|$ *with angle constraint*

B. Two-Bus Network with Bus Constraints (Fixed Voltage Magnitudes)

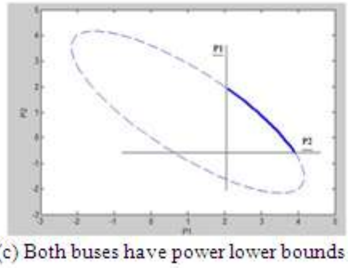
Considering the effect of bus power constraints on the two-bus network, there are three possible cases for the injection region [8] as it shown in figure 4.

In figure 4(a), both buses have power upper bounds. In figure 4(b), P_1 has power upper bounds, P_2have both power upper and lower bounds. For these two situations, the Pareto-fronts of bold curves are the same as the Pareto-fronts of convex hull. This means the non-convex problem can be translated to an easy-to-solve convex optimization problem.

In figure 4(c), both buses have power lower bounds. In this case, the Pareto-front of bold curve is no longer the same as that of the convex hull. This reveals that if two buses are connected, then they cannot be simultaneously has a tight bus active power lower bound.



(a) Both buses have power upper bounds (b) P_1 has upper bounds, P_2 have both upper and lower bounds



(c) Both buses have power lower bounds

Figure 4. Three possible cases for the bus power constrained injection region.

C. Two-Bus Network With Angle, Thermal and Flow Constraints (Variable Voltage Magnitudes).

Case *A* and *B* assume that the bus voltage magnitudes $|V_1|$ and $|V_2|$ are fixed. In this part, the goal is to study the behavior of the two-bus network in figure 1 when the bus voltage magnitudes vary in a small range. We try to simulate this case in Matlab to check the geometry characteristic of this model.

Use structure in figure 1, equation (10) and (11) still applies to this model, assume that $V_1=|V_1|\exp(j\theta_1)$, $V_2=|V_2|\exp(j\theta_2)$, $0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$, θ_1 and θ_2 's range depend on realistic constraints.

Take b=5, g=1, the new formulas turned to be

$$P_1 = |V_1|^2+5|V_1||V_2|\sin(\theta)- |V_1||V_2|\cos(\theta) \quad (16)$$

$$P_2 = |V_1|^2- 5|V_1||V_2| \sin(\theta)- |V_1||V_2|\cos(\theta) \quad (17)$$

$$0.9 \leq |V_1| \leq 1.1 \quad (18)$$

$$0.9 \leq |V_2| \leq 1.1 \quad (19)$$

The result is a series of hollow ellipse, as it is shown in Figure 5.

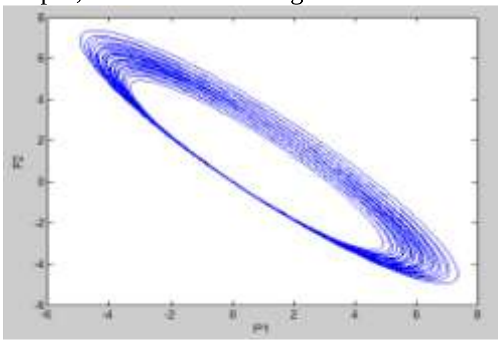


Figure 5. The region corresponding to b=5, g=1, $0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$

Figure 5 reveals the relationship of P_1 and P_2 with only voltage constraints. To see clearly what is happening on the bound, limit the number of hollow ellipses to 4 by adjusting $|V_1|$ and $|V_2|$, The result is shown in figure 6. It is obvious that the hollow ellipses at each voltage intersect with each other.

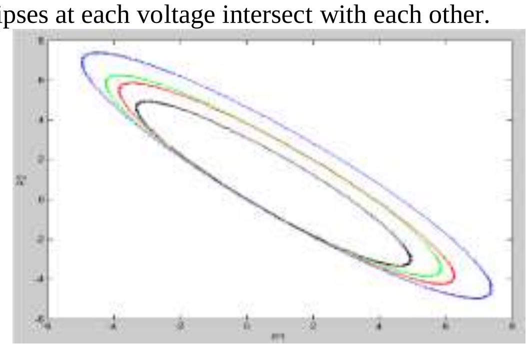


Figure 6. The region corresponding to b=5, g=1, $0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$, with only four ellipses.

Then try to fix $|V2|$=1, only change $|V1|$ between 0.9 and 1.1, other conditions remain the same with Figure 5. The result is shown in figure 7. It is interesting that the curves below the major axis intersect with each other, but the curves above the major axis have no intersections.

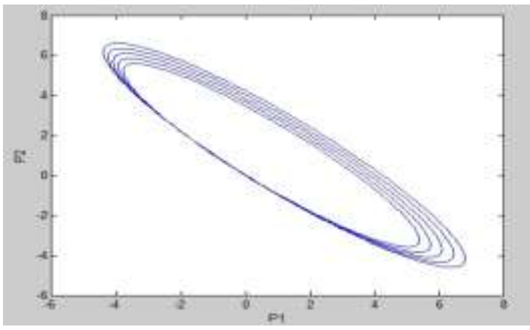


Figure 7. The region corresponding to b=5, g=1, $0.9 \leq |V_1| \leq 1.1$, $|V_2| = 1$

Try to change the b/g ratio to see its effect on the bounds. Take b=3, g=1, other conditions remain the same with

Figure 5.. The result is shown in figure 8.

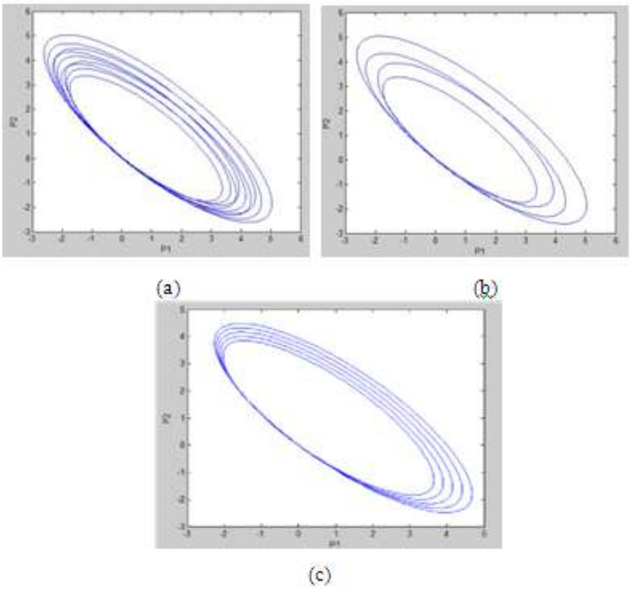


Figure 8. The region corresponding to b=3, g=1, (a) is under the condition

$0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$ with multiple ellipses. (b) is under the condition
$0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$ with only four ellipses. (c) is under the condition
$0.9 \leq |V_1| \leq 1.1$, $|V_2| = 1$

As we can see, the change of *b/g* ratio only changes the length of major axis/ length of minor principle axis ratio. The main properties of the problem haven't change.

The above cases in IV.C only consider voltage constraints. We then consider the effect of thermal, line flow and angle constraints on this problem.

In terms of the line parameter b_{12} and g_{12}, the angle thermal, line flow and angle constraints can be written as [9]

$$-\tan^{-1}\left(\frac{b_{12}}{g_{12}}\right) < \underline{\theta}_{12} < \overline{\theta}_{12} < \tan^{-1}\left(\frac{b_{12}}{g_{12}}\right) \quad (20)$$

To simulate this case in Matlab, we take b=5, g=1, then the angle is between -1.3734 and 1.3734.. The result is shown in figure 9.

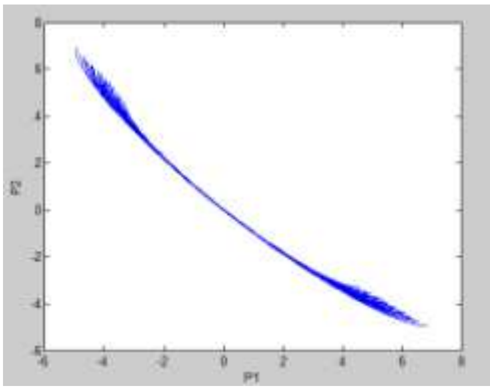


Figure 9. The region corresponding to b=5, g=1, $0.9 \leq |V_1| \leq 1.1$, $0.9 \leq |V_2| \leq 1.1$ with angle constraints.

From the above observation, we see the corresponding region of different cases and get a general ideal of the problem. To get a more strict deduction, we do the following theory analysis in Section V.

V. Theory Analysis Of Two-Bus Network With Variable Voltages

To simplify this problem, take b=5, g=1, equation (16) and (17) can also be written as

$$\left(\frac{P_1+P_2-V_1^2-V_2^2}{2V_1\times V_2}\right)^2 + \left(\frac{P_1-P_2-V_1^2+V_2^2}{10V_1\times V_2}\right)^2 = 1 \quad (21)$$

Letting $V_1^2 = X_1$, $V_2^2 = X_2$, assume the bound of V_1 and V_2 is [a, b], the bound of P_1 is [c, d], and the bound of P_2 is [e, f].

Then equation (21) can be written as

$$\frac{(P_1+P_2-X_1-X_2)^2}{4} + \frac{(P_1-P_2-X_1+X_2)^2}{100} = X_1 X_2 \quad (22)$$

The original OPF problem for two-bus networks with variable voltages can be simplified as below.

Minimize: $f(P_1, P_2)$ (23) Subject to:

$g(x) \le 0$ (24)

$X_1 - b^2 \le 0$ (25)

$a^2 - X_1 \le 0$ (26)

$X_2 - b^2 \le 0$ (27)

$a^2 - X_2 \le 0$ (28)

$P_1 - d \le 0$ (29)

$c - P_1 \le 0$ (30)

$P_2 - e \le 0$ (31)

$f - P_2 \le 0$ (32)

where $g(x) = 25\,(P_1 + P_2 - X_1 - X_2)^2 + (P_1 - P_2 - X_1 + X_2)2 - 100X1X2$ (33)

We have already known that f is an increasing function. Therefore, for the above model, if $g(x)$ is a convex function, then this problem will turn to be a convex optimization problem, which is easy to solve.

A. Function g only has two variables. The bus active power P_1 and P_2 have upper and lower bounds, the voltage magnitudes are fixed.

Write the Hessian matrix of function g.

$$H(g(P_1, P_2)) = \begin{bmatrix} \frac{\partial^2 g}{\partial P_1^2} & \frac{\partial^2 g}{\partial P_1 \partial P_2} \\ \frac{\partial^2 g}{\partial P_2 \partial P_1} & \frac{\partial^2 g}{\partial P_2^2} \end{bmatrix} = \begin{bmatrix} 52 & 48 \\ 48 & 52 \end{bmatrix} \quad (34)$$

It is obvious that $H(g(P_1, P_2))$ is positive definite, then $g(P_1, P_2)$ is a convex function. Therefore, the OPF problem is a convex optimization problem for two-bus network, if the voltage magnitudes are fixed, and bus active power have upper and lower bounds.

B. Function g has three variables. The bus active power P_1 and P_2 have upper and lower bounds. One of the voltage magnitudes ($|V_2|$) is fixed, another ($|V_1|$) is variable.

Write the Hessian matrix of function g. Note that Eig means the eigenvalue of a matrix.

$$H(g(P_1, P_2, X_1)) = \begin{bmatrix} \frac{\partial^2 g}{\partial P_1^2} & \frac{\partial^2 g}{\partial P_1 \partial P_2} & \frac{\partial^2 g}{\partial P_1 \partial X_1} \\ \frac{\partial^2 g}{\partial P_2 \partial P_1} & \frac{\partial^2 g}{\partial P_2^2} & \frac{\partial^2 g}{\partial P_2 \partial X_1} \\ \frac{\partial^2 g}{\partial X_1 \partial P_1} & \frac{\partial^2 g}{\partial X_1^2} & \frac{\partial^2 g}{\partial X_1 \partial P_2} \end{bmatrix} = \begin{bmatrix} 52 & 48 & -52 \\ 48 & 52 & -48 \\ -52 & -48 & 52 \end{bmatrix} \quad (35)$$

$$\text{Eig}\,[H(g(P_1, P_2, X_1))] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5.0389 & 0 \\ 0 & 0 & 150.6911 \end{bmatrix} \quad (36)$$

From (36) we know that $H(g(P_1, P_2, X_1))$ is positive semi-definite, then $g(P_1, P_2, X_1)$ is a convex function. Therefore, the OPF problem is a convex optimization problem for two-bus network, if only one of the voltage magnitudes is fixed, and bus active power have upper and lower bounds,

C. Function g has four variables. The bus active power P_1, P_2 and the voltage magnitudes $|V_1|$, $|V_2|$ all have upper and lower bounds.

Write the Hessian matrix of function g.

$$H(g(P_1, P_2, X_1, X_2)) = \begin{bmatrix} \frac{\partial^2 g}{\partial P_1^2} & \frac{\partial^2 g}{\partial P_1 \partial P_2} & \frac{\partial^2 g}{\partial P_1 \partial X_1} & \frac{\partial^2 g}{\partial P_1 \partial X_2} \\ \frac{\partial^2 g}{\partial P_2 \partial P_1} & \frac{\partial^2 g}{\partial P_2^2} & \frac{\partial^2 g}{\partial P_2 \partial X_1} & \frac{\partial^2 g}{\partial P_2 \partial X_2} \\ \frac{\partial^2 g}{\partial X_1 \partial P_1} & \frac{\partial^2 g}{\partial X_1 \partial P_2} & \frac{\partial^2 g}{\partial X_1^2} & \frac{\partial^2 g}{\partial X_1 \partial X_2} \\ \frac{\partial^2 g}{\partial X_{12} \partial P_1} & \frac{\partial^2 g}{\partial X_2 \partial P_2} & \frac{\partial^2 g}{\partial X_2 \partial X_1} & \frac{\partial^2 g}{\partial X_2^2} \end{bmatrix} = \begin{bmatrix} 52 & 48 & -52 & -48 \\ 48 & 52 & 48 & -52 \\ -52 & 48 & 52 & -52 \\ 48 & -52 & -52 & 52 \end{bmatrix} \quad (37)$$

$$\mathrm{Eig}\,[H(g(P_1, P_2, X_1X_2))] = \begin{bmatrix} -61.8034 & 0 & 0 & 0 \\ 0 & 3.8403 & 0 & 0 \\ 0 & 0 & 104.1597 & 0 \\ 0 & 0 & 00 & 161.8034 \end{bmatrix} \quad (38)$$

From (38) we know that $H(g(P_1, P_2, X_1X_2))$ is indefinite, $g(P_1, P_2, X_1X_2)$ is neither a convex function nor a concave function. Therefore, the OPF problem is no longer a convex optimization problem for two-bus network, if all the voltage magnitudes and bus active power are variable.

VI. Futrue Work

For future work, we can explore the possibility of using SDP method to solve the OPF problem.

The general idea of Semidefinite Programming (SDP) method is solving the Lagrangian dual problem instead of solving the OPF problem directly [12]- [13]. In [5], a SDP has been proposed, of which the dual is a convex relaxation of the OPF problem. Since both the SDP and the rank relaxation are convex, duality holds between them, which suggest that the duality gap between OPF and its SDP is zero. Therefore, a global optimal solution can be extracted from the optimal solution of its Lagrangian dual. Paper [5] provides a sufficient condition under which the existence of no duality gap for the OPF problem is guaranteed. In paper [14], it is proved that if the loads are over-satisfied, then the duality gap for OPF over a tree network is always zero, which implies the OPF problem can be solved. In paper [10], it is proved that every OPF problem is guaranteed to be solvable in polynomial time after two modifications (a. every loop of the network has a line with a phase shifter; b. load over-satisfaction is allowed).

Compared with the study on two-bus network in previous sections, analyzing by SDP method can solve more general OPF problem and can reveal more underlying system properties.

VII. Conclusion

This paper discusses the power system optimization over tree (radial) networks, which is the most commonly topology in distribution systems. Since the optimal power flow (OPF) problem is non-convex and challenging, the main purpose of this paper is to investigate whether the non-convex region preserves important properties of a convex set under variable voltage magnitudes. Simulation has been done in this paper and theory analysis shows that in a two-bus network with bus active power constraints, if there is one or more voltage magnitudes is fixed, then the OPF problem can be translated to a convex optimization problem, which is easy to solve. In the end of this paper, SDP method has been introduced to solve the OPF problem from another sight.

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